Persistency for Higher-Order Pseudo-Boolean Optimization

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ABSTRACT

A pseudo-Boolean function is a function from a 0/1-vector to the reals. Minimizing pseudo-Boolean functions is a very general problem with many applications. In image analysis, the problem arises in segmentation or as a subroutine in task like stero estimation and image denoising. Recent years have seen an increased interest in higher-degree problems, as opposed to quadratic pseudo-Boolean functions.

A method for minimizing quadratic pseudo-Boolean functions approximately is roof duality [1]. One of its important properties besides providing a lower bound is persistency: the fact that the approximation gives a partial solution of 0, 1, and "unknown." This is a certificate that there exists a solution to the original problem equal to the partial solution for all known variables.

Higher-order pseudo-Boolean functions can always be reduced to the quadratic case by the introduction of additional variables [1]. This is not always satisfactory, for example, the approach in [1] introduces non-submodular terms of very high magnitude.

This paper shows that the approach for quadratic functions originally based on vertex packing [2,3] readily carries over to pseudo-Boolean functions of higher degree.

BODY

A pseudo-Boolean function is a posiform [1]. Write its maximization as vertex packing. Solve a bipartite problem [3]; et voilà: persistency!

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