First-order theorem proving in the exploration of Andrews-Curtis conjecture

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ABSTRACT

Proving or disproving the Andrews-Curtis conjecture remains one of the outstanding open problems in combinatorial group theory. In short, it states that every balanced presentation of the trivial group can be transformed (aka. AC-simplified) into a trivial presentation by a sequence of Nielsen transformations on the relators together with conjugations of relators. It is believed that the conjecture may well be false and there are several potential (series of) counterexamples for which required simplifications are not known. Specialized search algorithms, including genetic search algorithms have been used so far to refute the potential counterexamples, see recent overview in [1].

In this paper we propose an alternative approach and formulate the term rewriting system ACT for AC-transformations, and its translation(s) into the first-order logic. The problem of finding AC-simplifications is reduced to the problem of proving first-order formulae, which is then tackled by the available automated theorem provers. Our experiments show that for all proposed potential counterexamples where non-trivial AC-simplifications were found by specialised search algorithms, AC-simplifications can be found by the generic theorem proving too. That opens an opportunity to apply present and future developments in automated theorem proving to the exploration of Andrews-Curtis conjecture. Further details of our approach can be found in [2].

BODY

Potential counterexamples to Andrews-Courtis conjecture can be refuted efficiently by automated theorem proving.

REFERENCES

[1] J. Swan, G. Ochoa, G. Kendall, M. Edjvet (2012) Fitness Landscapes and the Andrews-Curtis Conjecture, Int. J. of Algebra and Computation, Vol. 2, No. 22, pp. 125009

[2] A. Lisitsa, Automated reasoning in the exploration of Andrews-Curtis conjecture, `http://www.csc.liv.ac.uk/~alexei/AC-conjecture/`

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